Decision Processes, THRALL, R. M., COOMBS, C. H., AND DAVIS, R. L., Editors, John Wiley and Sons, New York, 1954, viii + 332, $5.00.

Decision Processes, while nominally a book, is in fact a one-issue journal consisting of nineteen mathematical and experimental papers on statistical decision theory, game theory, learning theory, and measurement theory (including utility measurement)—all parts of an area well described by the title. The work stems from an eight-week summer conference on "The Design of Experiments in Decision Processes" held at the RAND Corporation in 1952. On the grounds that such a book will not be definitive and that research activity in the area is lively, the editors felt that "an informal and relatively speedy method of printing" was justified. While agreeing with their conclusion, two questions can be raised: Did these considerations actually force the publisher to employ such an unattractive format? And do not these same reasons, plus the desirability of the lowest possible price for a volume soon to be antedated, suggest paper, not cloth, covers?

The volume begins with an introduction by R. L. Davis, which outlines the area, cites a bit of its history, and sketches the major focus and results of each paper. A clear notion of the relevance of this book to one's interests can be obtained by reading these eighteen pages. The next article, also introductory in nature, "Some Views on Mathematical Models and Measurement Theory" by C. H. Coombs, Howard Raiffa, and R. M. Thrall is divided into two parts. The first offers a highly idealized scheme of scientific research with particular emphasis on the role of mathematical models. The second part, on measurement models, is presented as an exemplification of the general scheme; it should serve as a handy reference of possible scales which, by being more complete, supplements Stevens' widely known classification. Definitions and social science illustrations are given of transitive relation, partial order, weak order, lattice, vector space, etc.; the interrelations among them are discussed and neatly summarized in a diagram.

The remaining articles are grouped in four sections: individual and social choice, learning theory, theory and applications of utility, and experimental studies. Since it is impossible to discuss them all in detail, attention will be restricted to those the reviewer found particularly satisfying or stimulating; as it happens all four divisions of the book are represented.

L. A. Goodman's paper "On Methods of Amalgamation," John Milnor's "Games Against Nature," and the "Note on Some Proposed Decision Criteria" by Roy Radner and Jacob Marschak are all concerned with decision criteria for the selection of a strategy in a game against nature. Goodman offers a new criterion which, simultaneously, generalizes those of LaPlace, Bayes, and Copeland. Radner and Marschak present an example which suggests that both the Hurwicz generalization of the Wald minimax criterion and the Savage minimax regret criterion may be inadequate, and, as we shall see, Milnor's work raises similar doubts. The Hurwicz criterion leads to a decision distinctly at variance with common sense, and the Savage criterion depends on irrelevant alternatives, in a sense analogous to Arrow's usage. Milnor's paper, the most interesting and elegant of the three, overlaps the others, covering the LaPlace, Wald, Hurwicz, and Savage criteria. Milnor lists eleven axioms a criterion might meet, and he shows which are met by the four criteria mentioned, and which characterize each of the four. It is striking that all but the LaPlace criterion fail to meet a Pareto condition on strategies (domination), and that the LaPlace criterion fails on another axiom, which, while not so basic, seems desirable. Furthermore, no criterion can meet all eleven axioms, so one is led to consider classes of criteria defined by subsets of axioms which seem intuitively necessary. Milnor selects five as essential and three others as desirable; he shows that the class so defined is non-empty. Finding a simple characterization of this class of criteria, or indeed of any member of the class, remains an unsolved problem.

The first paper of part II, "A Formal Structure for Multiple-Choice Situations"

by R. R. Bush, Frederick Mosteller, and G. L. Thompson, is a welcome concise statement of the mathematical structure of the Bush-Mosteller stochastic learning model. As is well known, the model can be stated in very general terms, but most of the results and applications assume linear operators. A major and controversial part of the paper is an attempt by means of the "combining of classes and condition," to give a more respectable basis for this assumption than the intriguing observation that it works. Roughly, this condition requires that the model yield the same results whether or not two alternatives with the same set of outcome probabilities are combined. At first glance this seems to have the same status and intuitive necessity as, say, the requirement that the laws of physics shall be independent of the position of the observer; to the extent it has this status and necessity it is exciting. Careful inquiry, however, suggests otherwise, for the probabilities relating outcomes to alternatives are under the arbitrary control of the experimenter; hence, the model must allow for *any* possible combining of classes. It appears to the reviewer that this is too demanding to be considered intuitively necessary, and thus is not really a justification for the linearity assumption. Still a persuasive justification is needed, for the linear model fits an impressive collection of data. An example of such data is presented in "Individual Behavior in Uncertain Situations: An Interpretation in Terms of Statistical Association Theory" by W. K. Estes.

Part III, on utility, includes two papers on the existence of utility functions; these papers are interesting but mathematically the most difficult in the book. The first, "Representation of a Preference Ordering by a Numerical Function" by Gerard Debreu, is concerned with topological conditions on a weakly-ordered set which are sufficient to insure the existence of a utility function. If certain sets are closed, he shows that either separability and connectedness or perfect separability are sufficient. No algebra of probability-combining is assumed as in the von Neumann and Morgenstern theory, but no unique results are obtained. In "Multidimensional Utilities" Melvin Hausner examines the effect of dropping the Archimedean axiom from the von Neumann and Morgenstern axioms. Let ApB denote a probability combination of A and B; the axiom requires that if A is preferred to B, and B to C, then ApC and B are indifferent for some p. The possible objection to the axiom is seen when one lets A = five cents, B = two cents, and C = death. Hausner obtains the elegant results that any non-Archimedean "mixture" space satisfying the other von Neumann and Morgenstern axioms can be imbedded in an ordered vector space, and that any ordered vector space is lexicographically ordered in some basis. Some interesting applications of this theory are suggested by R. M. Thrall in "Application of Multidimensional Utility Theory."

"Towards an Economic Theory of Organization and Information" by Jacob Marschak initiates a fascinating normative study of decision-making by communicating "teams," where teams are defined to be groups with identical individual and group utility functions. A team may collect data, transmit information over a communication network at some cost, and take actions based on a decision rule. Three classes of problems are considered for a team which completes all observation before making any decisions. 1) *Procedural*: given a network and cost of communication, to select the best rules for governing information transmission and actions. 2) *Network*: given rules and a cost function over networks, to select the best communication network. 3) *Constitutional*: to select the best procedural-network pair. Several simple special cases are solved, but as Davis notes (p. 13): "The relatively difficult manipulations required even for these simple cases show for one thing how desirable further development and simplification of the theory would be, while on the other hand they serve to emphasize how difficult would be any analysis at all without the machinery of this formalization."

In the final experimental section, two of the four papers deal with coalition formation in the game-theory sense; both emphasize that psychological rather than "objective"

utilities are necessary for a descriptive theory. In "Tendencies Toward Group Comparability in Competitive Bargaining," Paul Hoffman, Leon Festinger, and Douglas Lawrence employ Festinger's psychological theories of group behavior to predict that those who are perceived as superior in an ability relevant to the conflict of interest involved tend to be excluded from effective bargaining. The confirming experiment was based on a symmetric 3-person game. One player was always a stooge who, in one variation, appeared to be of similar intelligence to the subjects, but who, in the second variation, was evidently of superior ability. In the latter case he was excluded from coalitions more often than in the former, the degree increasing with the importance subjects placed on the game situation. These results strongly suggest that utility functions are subject to modification by psychological manipulations—an unfortunate complication. More directly related to game theory itself is the paper "Some Experimental n-Person Games" by G. Kalisch, J. W. Milnor, J. Nash, and E. D. Nering. Several n-person games ($n = 4, 5, 7$) were run in characteristic function form, i.e., payments were stated for each possible coalition. In each case subjects bargained for 10 minutes, and they reported their agreements to an umpire who enforced them. Considering the rationality assumptions of the theory, the time limit seems questionable. The principal results appear to be: contrary to theory, strategically equivalent games were treated differently; the Shapley value tended to be similar, though by no means identical, to the experimental payments; no satisfactory method was devised to check the von Neumann and Morgenstern theory of solutions. If the authors intended to show that objective payments rather than subjective utilities are sufficient for descriptive purposes, the first result is most disturbing. The failure of the subjects to respond to the objective situation is further confirmed by the authors' observation that the subjects tended to form coalitions having large payments without regard to benefits resulting from other *apparently* less impressive coalitions. While the prospects of positive findings are not great, the experiment probably should be replicated under more carefully controlled conditions and using many more subjects. At that time data could be collected from the subjects prior to each run as to their perceptions of relative coalition strength per coalition member. We do not expect these to be the same as the "rational" ordering derived from the objective characteristic function, but it might be possible to establish that their bargaining behavior is consistent with their orderings. Certainly these two experiments reinforce the contention of von Neumann and Morgenstern that an individual's utility function need not be simply related to any objective measure arising from the situation.

In summary, we may agree with the editors that the book is not definitive and yet recommend it as stimulating and useful for those working in the area. Anyone attracted by any one of the papers will surely be interested in several others, and he may very well have a passing curiosity about most of them.

Center for Advanced Study in the Behavioral Sciences *R. Duncan Luce*